Issue of the *Annals of Econometrics* on
INDIRECT ESTIMATION METHODS IN FINANCE AND ECONOMICS
INTRODUCTION

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This special issue is the outcome of the small and high-powered conference on *Indirect Estimation Methods in Finance and Economics*. The conference took place at Abbey Hegne in Allensbach (Lake Constance, Germany) on May 2014. It commemorated the 20th anniversary of the seminal papers on Indirect inference of Gouriéroux, Monfort and Renault (1993), Simulated quasi-maximum likelihood of Smith (1993) and the Efficient methods of moments of Gallant and Tauchen (1996).

A CONSICE BACKWARD LOOKING TESTIMONIAL

We asked the authors of the three articles to comment on the origins and motivation for their research. We are thankful that they accepted without reservations. Their comments contain very interesting insights on how and why indirect methods appeared. Four aspects arise in all the comments. The first is that behind their novel econometric technique, there was the quest for answering to open economic and financial questions, which fully frames within the Ragna Frisch original definition of econometrics (Frisch, 1933). The second common aspect is that the authors build from past research, namely from inference under model misspecification and the notion of pseudo-true value. Third, the authors understood from the beginning that the so-called binding function that connects the model of interest with the auxiliary model played a central role in indirect methods, and that simulations were needed. Last, all the authors also look at the future, which naturally dovetails with this special issue. In the sequel we show excerpts of their texts regarding these four common aspects.

<u>On the motivation</u>

For Smith the motivation was macro oriented:

The basic ideas of indirect inference, as it later became known, grew out of Chapter 2 of my 1990 dissertation at Duke University under the supervision of John Geweke (Smith, 1990) […] They began with an effort to conduct Bayesian inference for micro-founded macroeconomic models, such as real-business-cycle (RBC) models […] The initial idea—inspired in part by early work on simulation estimation by Lee and Ingram (1991) and Duffie and Singleton (1993) - was to construct an approximate likelihood for such a model by: one, computing a nonlinear solution to the model; two, simulating time-series data from it; three, fitting a vector autoregression (VAR) to the simulated data; and, four, inserting the estimated VAR parameters into the likelihood associated with the VAR. This approach avoided making use of possibly inaccurate nonlinear solutions to the structural model, while at the same time it provided a theory of the error term in the VAR as arising in part from fitting a linear VAR to a nonlinear data-generating process.

While for Gouriéroux, Monfort and Renault the motivation was financial:

The principle of indirect inference was initially introduced to solve a major financial issue when pricing derivatives in practice, i.e. the need to find consistent estimators of a parametric diffusion equation with daily observations only.

And for Gallant and Tauchen the motivation was both macro and financial

At the time EMM was developed, the developers and their co-authors were engaged in several projects of model specification analysis with focus on the determination of plausible macro models and plausible continuous time models for financial market dynamics.

On building from past research

As mentioned, indirect methods appeared as natural theoretical continuation from past research, namely previous works on misspecification and the notion of pseudo-true value. For Smith:

This approach to simulation estimation—using the parameters of a misspecified "model of the model" as a vehicle for the estimation of underlying structural parameters—appeared, in fact, to be a quite general and legitimate approach to classical estimation. […] I dubbed this first approach to indirect inference "simulated quasi-maximum likelihood" because it maximizes an approximate likelihood, subject to the restrictions imposed on its parameters by the structural model […] I benefitted *from White (1982) work on maximum likelihood estimation of misspecified models […] and from Gourieroux, Monfort, and Trognon (1984) that had shown that pseudo likelihoods could in some circumstances deliver consistent and robust parameter estimates.*

Similarly, Gouriéroux, Monfort and Renault also used their past research on QMLE and White's:

This principle [of indirect inference] was a natural continuation of previous works on misspecified models developed in the eighties (e.g. Gourieroux, Monfort, and Trognon, 1984). In analyzing pseudo-maximum likelihood approaches, misspecified parametric models were used to provide consistent estimators of the parameters in semi-parametric models and the notion of pseudo-true value introduced by T., Sawa and H., White was intensively used.

Also for Gallant and Tauchen, the use of SNP models brought the aspect on misspecification:

Simulated method of moments first surfaced in econometrics around 1985-1990, and the developers were urged by Lars Peter Hansen to think about using the nonlinear semi-parametric SNP models as a standard for comparing simulated data to observed data

On the biding function

A third common denominator is the use of the binding function, a term coined in the indirect inference article of Gouriéroux, Monfort and Renault. For Smith, the binding function was the "vehicle" connecting both the model of interest and the auxiliary model:

This approach to simulation estimation—using the parameters of a misspecified "model of the model" as a vehicle for the estimation of underlying structural parameters—appeared, in fact, to be a quite general and legitimate approach to classical estimation, with wide-ranging applications

While for Gouriéroux, Monfort and Renault

Simulated binding functions were the basis for defining consistent estimation methods for the problem of diffusion equation. Indeed, the trajectory of a diffusion process is easily simulated with a good accuracy and a Euler discretization can be used as the misspecified instrumental model. This example showed the importance of selecting an appropriate instrumental model, generally derived from a structural model by various kinds of simplifications.

And for Gallant and Tauchen

The approach of re-estimating the binding function on the fly for each simulation was immediately dismissed as totally impractical in the nonlinear case. On the other hand, the practicality of mimicking the first order conditions, instead of the estimation, seemed quite evident. It was also evident that the scores of a sieve would be the best choice of moment conditions for such applications: spanning implies statistical efficiency and scores with analytical representation implies computational efficiency.

<u>On the future of indirect methods</u>

All the authors agree that there is a large window of opportunities and research on indirect methods, both theoretical and applied. For Smith:

Despite its practical success in applied work, one way in which indirect inference has perhaps not yet lived up to its full potential, despite some work in this direction by Dridi and Renault (1998) and Dridi, Guay, and Renault (2007), is in exploiting its ability to use different windows to select aspects of the data on which to focus. I always viewed this flexibility as a distinguishing feature of indirect inference. My dissertation pointed out that some windows might be better at discriminating between structural models than others, but did not develop these ideas much further. The goal here is to use indirect inference […] to find a middle way between the generalized method of moments and full structural estimation which does permit such analysis but also introduces many incidental parameters. […] Future research will need to develop systematic methods for choosing auxiliary models that discriminate between structural models in ways that matter for answering questions in economics.

While for Gouriéroux, Monfort and Renault

The idea of using an instrumental model has been recently extended to the simultaneous use of several instrumental models (Gourieroux and Monfort, 2017), each of them capturing partial features of the structural model, and the optimal combination of these instrumental models leads to the Composite Indirect Inference method. Indirect inference and its various extensions are especially convenient for complicated models with intractable likelihood functions or moments, such as continuous time models, high dimensional dynamic models with latent factors, or networks. New applications are regularly proposed in several fields like credit scoring, biometrics, machine learning, data compressing and even in unexpected domains like the organization of econometric exams for the students.

And for Gallant and Tauchen

[…] the Bayesian component of the exact Bayesian EMM estimator adapts ideas presented in Gallant (2016) published with commentary. As for the EMM component, five features distinguish the EMM simulation estimator as customarily implemented with the SNP (seminonparametric) auxiliary model in a frequentist context: efficiency in large samples, tests for relative structural model performance,

tests for absolute model performance, t-tests on scores that suggest the cause of model failure, and an auxiliary model selection protocol. Applications dictate the importance of these five features

WHAT IS IN THIS SPECIAL ISSUE

Since its origin (Smith, 1990), Indirect Inference has been used for the estimation of complicated dynamic structural models, as DGSE models as well as intertemporal asset pricing models. Inference in these models is especially challenging, not only due to their high degree of non-linearity, but also due to misspecification or weak identification issues. While Dridi, Guay and Renault (2007) have pointed out the tight connection between indirect inference in these circumstances and the practice of calibration of dynamic structural models, the first two papers of this special issue push this argument even further.

Grammig and Kuchlin revisit this tight connection in the context of asset pricing models with long-run risk. Two recent contributions to the indirect inference literature have set a special focus on the challenge of indirect inference in these models. While Calvet and Czellar (2015) have recently discussed a specific approach, based on a simplified misspecified model, Gospodinov, Komunjer and Ng (2017) have explicitly devised an indirect estimator that is consistent despite misspecification of part of the model. In *A two-step indirect inference approach to estimate the long-run risk asset pricing model,* Grammig and Kuchlin go one step further by explicitly hedging against the risk of misspecification by a neat two-step approach to indirect inference. By disentangling the estimation of the model's macroeconomic dynamics and the investor's preferences, they can devise an efficient, albeit robust to partial misspecification, indirect estimation of these models. It allows them to provide a critical re-assessment of the long-run risk model's ability to reconcile the real economy and financial markets.

Blasques and Duplinskiy start from the remark that Bayesian methods have become increasingly popular as a tool for conducting inference in structural models since priors offer a way to exert control over the estimation results, which especially matters in case of misspecification or weak identification. In *Penalized Indirect Inference* They propose a penalized indirect inference, which entails a frequentist estimation strategy that allows them to exert some control on the estimation results. Then they can derive the asymptotic properties of the penalized indirect inference estimator for both correctly and misspecified models, as well as model with some identification issues. The advantages of the penalized approach are illustrated by an empirical study of a DSGE model.

Another feature of structural econometric models is also to include often exogenous variables. Gourieroux, Monfort and Renault (1993) had devised an extension of the indirect inference theory to deal with exogenous variables: they cannot be simulated but the simulated values of the endogenous variables should be drawn from their conditional probability distribution given the observations on exogenous variables, However, while Altonji, Smith and Vidangos (2013) have recently noted that indirect inference provides a straightforward way to deal with missing data ("after generating a complete set of simulated data, one simply omits observations in the same way they are omitted in the observed data"), this idea cannot work in the case of structural models with missing data on exogenous variables. There is no way to simulate given missing observations on exogenous variables. In case of endogenously missing exogenous variables, a naive application of indirect inference will lead to a selection bias. In *Indirect Inference with Endogenously Missing Exogenous Variables*, Chaudhuri, Frazier and Renault provide a solution in the case of exogenous variables that can be missing at random (MAR) endogenously. By inverse probability weighting (IPW) the "complete case" observations, i.e., sample units with no missing variables for the observed and simulated samples, they propose a new method of indirect inference to consistently estimate the structural model of interest. Asymptotic properties of the new estimator and its efficiency are discussed and illustrated by a Monte Carlo exercise in the context of a multinomial Probit model.

Dovonon and Hall propose an extension of the asymptotic theory of indirect inference in an additional case of identification weakness. While in linear models, first-order local and global identification are identical, non-linear models pave the way for cases where first order local identification may fail while global identification is granted. One interesting case is when local identification holds at second but not first order. Sargan (1983) and Rotnitzky, Cox, Bottai and Robbins (2000) have devised an asymptotic distributional theory for estimators obtained respectively by IV in a nonlinear in parameters model and Maximum Likelihood. Dovonon and Renault (2019, 2013) have developed a general theory for GMM inference with a special focus on multivariate GARCH models with latent factors. In *The Asymptotic Properties of GMM and Indirect Inference under Second-order Identification,* Dovonon and Hall extend the GMM inference to the case of indirect inference. The limit distributions are shown to be non-standard but it is shown that they can be easily simulated, making it possible to perform inference about parameters in a general setting of indirect inference with second-order identification.

So-called Approximate Bayesian Computation (ABC) has received considerable attention in the econometrics literature in recent years. While originally developed for inference in genetics models, it is now widely used for computation of Bayesian posterior distributions in models where the likelihood is not available on closed form. In *The ABC of Simulation Estimation with Auxiliary Statistics*, Jean-Jacques Forneron and Serena Ng link together ABC with indirect inference and provide new results on implementation of ABC and derive new results that compare the properties of ABC with corresponding indirect inference estimators. Specifically, they derive a novel sampler for the implementation of ABC, and demonstrate through a stochastic expansion that the resulting estimator enjoys attractive properties.

Ron Gallant and George Tauchen also take a Bayesian approach in their work on *Exact Bayesian Moment Based Inference for the Distribution of the Small-Time Movements of an Ito Semimartingale* where they propose a general procedure for conducting Bayesian inference based on the efficient methods of moments. They apply the general method to the problem of conducting inference of the jump distribution in high-frequency financial data. They demonstrate how their method can help identifying these jumps and provide a better understanding of their behaviour. In their empirical application, they find that pure continuous-time jump-like models play an important role in our understanding of high-frequency asset price fluctuations.

Liang Jiang, Xiaohu Wang and Jun Yu also consider jumps in continuous-time models, but here the jump arises from structural breaks. They develop *New Distribution Theory for the Estimation of Structural Break Point in Mean* by formulating the problem in a continuous-time setting. The theory provides new insights into the behaviour of standard break point tests in a discrete-time setting, and allows them to provide tools for more precise inference in this setting.

As is well-known, simulation-based estimators of discrete-choice models suffer from discontinuities which hamper both the practical implementation and theoretical analysis of these. In *Generalized*

Indirect Inference for Discrete Choice Models, Marianne Bruins, James A. Duffy, Michael P. Keane, and Anthony A. Smith, Jr. propose a novel simulation procedure that smoothes out the kinks in the simulations and thereby restores differentiability of the objective function defining the indirect inference estimators. This comes at the price of a small additional bias due to the smoothing, but in return the estimator is computationally much more efficient. This is demonstrated both theoretically and numerically in the paper.

R. Golombek and A. Raknerud also deal with the discontinuity of the simulated trajectories of discrete-continuous choice models in the article *Exist Dynamics of Starts-up Firms: Structural Estimation using Indirect Inference* when applying EMM to estimate dynamic structural models for the exit and investment of firms under uncertainty with application to Norwegian data for start-ups. They propose a multi-step procedure to smooth the simulated trajectories based on averaging, discretization and bilinear interpolation with small costs on the magnitude error of the approximated, but differentiable binding function. For their purposes, they use an auxiliary model with the same parameters as the structural one to facilitate identification, which is estimated by QML in a multi-step framework.

Identification is also an issue of the article *Misspecification of Noncausal Order in Autoregressive Processes* that examines the potential effects of misspecified estimated noncausal order in causal and mixed autoregressive models. C. Gourieroux and J. Jasiak use biding functions, which are at the centre of the indirect inference estimation techniques, to characterize the misspecification of constrained maximum likelihood estimators of the autoregressive parameters of a causal model estimated from data generated by purely noncausal or mixed causal-noncausal models. The authors show the usefulness of binding functions also in deriving the asymptotic properties of the unconstrained maximum likelihood estimators of the noncausal order and in developing direct and indirect encompassing tests for the robust identification of the noncausal order.

The challenge of estimating general dynamic latent factor models has been for almost two decades at the centre of interest of a large body of the econometrics literature. G. Fiorentini, A. Galesi and E. Sentana provide in the article *A Spectral EM Algorithm for Dynamic Factor Models* a fast-iterated indirect inference procedure to estimate such models with ARMA structures in the latent components. The procedure, which involves auxiliary sequential OLS regressions is developed to ease the computational burden of an EM algorithm in the frequency domain also proposed in the paper. Moreover, the procedure turns out to be very efficient (the estimators coincide with the ML ones in the limit) and, thus, overcomes the efficiency loss of the existing indirect inference approaches using approximated auxiliary models.

Another contribution in this direction is provided by G. Calzolari and R. Halbleib in the article *Estimating Stable Latent Factor Models by Indirect Inference*, who also apply an indirect inference approach involving sequential auxiliary estimation to estimate dynamic latent factors with multivariate stable distributed factors and idiosyncratic noises. They implement an auxiliary approach that consists first in extracting static factors and then applying on them the same dynamics as in the structural model and a Student's distribution. The sequential auxiliary approach proves again its value in terms of computational and efficiency gains compared to the ones used in standard indirect inference techniques when estimating general dynamic factor models.

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ORDER OF THE PAPERS IN THE SPECIAL ISSUE:

1. A two-step indirect inference approach to estimate the long-run risk asset pricing model

2. Penalized Indirect Inference

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4. The Asymptotic Properties of GMM and Indirect Inference under Second-order Identification

5. The ABC of Simulation Estimation with Auxiliary Statistics

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9. Exit dynamics of start-up firms: Structural estimation using indirect inference

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12. Estimating stable latent factor models by indirect inference